AC/04.08.2018/RS1

NAAC REACCREDITED "A" GRADE, CGPA 3.51/4.00
BEST COLLEGE AWARD - UNIVERSITY OF MUMBAI

SIES College of Arts, Science and Commerce

(Autonomous)

Affiliated to Mumbai University

Syllabus under Autonomy - July 2018

Program: T. Y. B.Sc.

Course: Mathematics

Choice Based Credit System CBCS

with effect from the academic year 2018-19

Program: T. Y. B.Sc.
Course: Mathematics

Broad Objectives:

To provide a degree programme in mathematics which is intellectually challenging and rigorous and whose graduates are well placed to pursue post graduate studies or to enter employment.
On successful completion of this course, all students should

1. have learnt to apply critical and analytical reasoning and to present logical and concise arguments.
2. have developed problem solving skills
3. have covered the core topics of advanced mathematics which our Department considers appropriate to their degree programme.
4. be able to comprehend high levels of abstraction in study of pure mathematics.

Learning Outcomes

Semester V

SIUSMAT51 : Multivariable Integral Calculus

This course is an extension of Integral calculus introduced in previous semester. In this course, students are introduced to Integral Calculus of several variables. They should be enabled to

- Understand the concept of Riemann Integral on a rectangle and a box.
- Apply Fubini's theorem for computation of integrals.
- Compute Areas, Volumes, Centre of Mass, using integrals.
- Understand the concept of parametric curve and surface.
- Compute line and surface integrals.
- Study the correlation between various integrals through Greens, Stokes and Gauss theorems.

SIUSMAT52 : LINEAR ALGEBRA

This course is an extension of Linear algebra introduced in second year. The concept of vector spaces is extended to quotient spaces. The course aims to enable the students to

- Understand quotient spaces,finding its dimension , orthogonal transformations,isometry.
- Cayley Hamilton theorem and its applications.
- Find eigenvalues and eigenvectors.
- Understand diagonalization and diagonalize the matrix.
- Understand orthogonal diagonalization, orthogonally diagonalize the matrix.

SIUSMAT53 : Topology of Metric Spaces

This course aims at introducing the students to the concept of metric space and to enable them to

- Discuss examples of Metric Spaces, Normed Linear Spaces
- Sketch Open Balls in R^2, classify Open and Closed sets, Equivalent Metrics
- Understand the concepts like Subspaces, Interior points, Limit Points, Dense Sets and Separability, Diameter of a set, Closure, and apply in problem solving.
- Discuss Sequences with respect to Boundedness and, Convergence.
- Understand the concept of Cauchy sequences and Complete Metric Spaces.
- Understand the concept of Compact Sets.

SIUSMAT54 : Number Theory and Its applications I

This course presents rigorous development of Number Theory using axioms, definitions, examples and Theorems. It aims to enable the students to

- Understand the basic structure and properties of integers.
- Improve their ability of mathematical thinking.
- Understand the Concepts such as secrecy, espionage, code cracking through real life examples.
- Understand and be able to use modern cryptographic methods .

SEMESTER V				
Multivariable Integral Calculus				
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT51	I	Multiple Integrals	2.5	3
	II	Line Integrals		
	III	Surface Integrals		
LINEAR ALGEBRA				
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT52	I	Quotient spaces and Orthogonal Linear Transformations	2.5	3
	II	Eigen values and Eigen vectors		
	III	Diagonalisation		
Topology of Metric Spaces				
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT53	I	Metric spaces	2.5	3
	II	Sequences and Complete metric spaces		
	III	Compact Sets		
Number Theory and Its applications I				
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT54	I	Congruences and Factorization	2.5	3
	II	Diophantine equations and their solutions		
	III	Primitive Roots and Cryptography		
PRACTICALS				
SIUSMATP5A	Practicals based on Courses **SIUSMAT51, SIUSMAT52**		3	6
SIUSMATP5B	Practicals based on Courses **SIUSMAT53, SIUSMAT54**		3	6

Teaching Pattern

1. Three lectures per week per course (1 lecture/period is of 48 minutes duration).

2. One practical of three periods per week per course (1 lecture/period is of 48 minutes duration).

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and examples.

SEMESTER V

Course : Multivariable Integral Calculus **Course Code: SIUSMAT51**

Unit I- Multiple Integrals (15 Lectures)

(i) Definition of double (resp: triple) integral of a bounded function on a rectangle (resp:box). Geometric interpretation as area and volume. Fubini's Theorem over rectangles and on any closed bounded sets, Iterated Integrals. Integrability and the integral over arbitrary bounded domains.

(ii) Integrability of the sums, scalar multiples, products of integrable functions.

(iii) Integrability of continuous functions. More generally, integrability of functions with a set of discontinuities of measure zero. (concept and examples only)

(iv) Domain additivity of the integral. Change of variables formula (Statement)

(v) Polar, cylindrical and spherical coordinates, and integration using these coordinates. Differentiation under the integral sign using Leibnitz Rule. Applications to finding the center of gravity and moments of inertia.

Unit II : Line Integrals (15 Lectures)

(i) Review of Scalar and Vector fields on R^n , Vector Differential Operators, Gradient, Curl, Divergence.

(ii) Paths (parametrized curves) in R^n (emphasis on R^2 and R^3), Smooth and piecewise smooth paths. Closed paths. Equivalence and orientation preserving equivalence of paths.

(iii) Definition of the line integral of a scalar and a vector field over a piecewise smooth path. Basic properties of line integrals including linearity, path additivity and behavior under a change of parameters. Examples.

(iv) Line integrals of the gradient vector field, Fundamental Theorem of Calculus for Line Integrals, Necessary and sufficient conditions for a conservative vector field .

(v) Green’s Theorem (rectangle). Applications to evaluation of line integrals.

Unit III : Surface Integrals (15 Lectures)

(i) Parameterized surfaces. Smoothly equivalent parameterizations. Surface Area.

(ii) Definition of surface integrals of scalar fields and of vector fields . Examples.

(iii) Curl , divergence of a vector field. Elementary identities involving gradient, curl and divergence.

(iv) Stokes’ Theorem (proof using Green's Theorem) , Examples.

(v) Gauss Divergence Theorem (proof only in the case of cubical domains). Examples.

Reference Books for Multivariable Calculus:

1. Apostol T. , (1969) , Second Edition, *Calculus, Vol. 2,* John Wiley, New York.
2. Courant R. and John F., (1989), *Introduction to Calculus and Analysis*, Vol.2, Springer Verlag, New York.
3. Ghorpade S. R. and Limaye B., (2009), *A course in Multivariable Calculus and Analysis*, Springer International Edition.
4. Marsden J.E. and Tromba A.J.,(1996) (Fourth Edition), *Vector Calculus*, W.H. Freeman and Co., New York.
5. Protter M.H. and Morrey C.B. Jr., (1995), *Intermediate Calculus*, Second Ed., Springer-Verlag, New York.
6. Stewart J. , (2009), *Calculus with early transcendental Functions*, Cengage, Learning.
7. Thomas G. B. and Finney R. L., 1998, *Calculus and Analytic Geometry*, Ninth Ed. (ISE Reprint), Addison Wesley, Reading Mass.
8. Widder D. V., (1989), *Advanced Calculus*, Second Ed., Dover Pub., New York.

Course: Linear Algebra Course Code: SIUSMAT52

Unit I. Quotient Spaces and Orthogonal Linear Transformations (15 Lectures)

Review of vector spaces over R , sub spaces and linear transformation.

Quotient Spaces: For a real vector space V and a subspace W, the cosets v + W and the quotient space V/W, First Isomorphism theorem of real vector spaces (fundamental theorem of homomorphism of vector spaces), Dimension and basis of the quotient space V/W , when V is finite dimensional.

Orthogonal transformations: Isometries of a real finite dimensional inner product space, Translations and Reflections with respect to a hyperplane, Orthogonal matrices over R , Equivalence of orthogonal transformations and isometries fixing origin on a finite dimensional inner product space, Orthogonal transformation of R^2 , Any orthogonal transformation in R^2 is a reflection or a rotation, Characterization of isometries as composites of orthogonal transformations and translation. Characteristic polynomial of an $n \times n$ real matrix. Cayley Hamilton Theorem and its Applications

Unit II : Eigenvalues and Eigenvectors (15 Lectures)

Eigenvalues and Eigenvectors of a linear transformation $T : V \to V$, where V is a finite dimensional real vector space and examples, Eigenvalues and Eigenvectors of nxn real matrices, The linear independence of eigenvectors corresponding to distinct eigenvalues of a linear transformation and a Matrix. The characteristic polynomial of an n real matrix and a linear transformation of a finite dimensional real vector space to itself, characteristic roots, Similar matrices, Relation with change of basis, Invariance of the characteristic polynomial and (hence of the) eigenvalues of similar matrices, Every square matrix is similar to an upper triangular matrix. Minimal Polynomial of a matrix, Examples like minimal polynomial of scalar matrix, diagonal matrix, similar matrix, Invariant subspaces.

Unit III: Diagonalisation (15 Lectures)

Geometric multiplicity and Algebraic multiplicity of eigenvalues of an nxn real matrix, Equivalence of the following statements:-

A is diagonalizable if and only if A has a basis of eigenvectors of A.

The sum of dimensions of eigenspaces of A is n if and only if the algebraic and geometric multiplicities of eigenvalues of A coincide, Examples of non diagonalizable matrices, Diagonalisation of a linear transformation $T : V \rightarrow V$, where V is finite dimensional real vector space and examples. Orthogonal diagonalisation and Quadratic Forms.Diagonalisation of real Symmetric matrices, Examples, Applications to real Quadratic forms, Rank and Signature of a Real Quadratic form, Classification of conics in R^2 and quadric surfaces in R^3.Positive definite and semidefinite matrices, Characterization of positive definite matrices in terms of principal minors. Applications of Diagonalisation.

Recommended Books:

1. Gopalkrishnan N.S ,(1994), *University Algebra,* Wiley Eastern Limited
2. Kumaresan S.,(2011), *Linear Algebra: A Geometric Approach,*PHI Learning Private Ltd.

Additional Reference Books

1. T. Bancho and J. Wermer,(2014), *Linear Algebra through Geometry*, Springer.

2. L. Smith,(1978), *Linear Algebra*,Springer.

3. M. R. Adhikari and Avishek Adhikari,(1999) *Introduction to linear Algebra,* Asian Books Private Ltd.

4. K Hoffman and Kunze, (1971),*Linear Algebra,* Prentice Hall of India, New Delhi.

5. Anton,(2010),*Linear Algebra and Applications.*John Wiley and Sons

Course: Topology of Metric Spaces Course Code: SIUSMAT53

Unit I: Metric spaces (15 Lectures)

Definition, examples of metric spaces R; R^2 ,Euclidean space R^n with its Euclidean, sup and sum metric, C (complex numbers), the spaces l^1 and l^2 of sequences and the space C[a; b], of real valued continuous functions on [a; b]. Discrete metric space.
Distance metric induced by the norm, translation invariance of the metric induced by the norm. Metric subspaces, Product of two metric spaces. Open balls and open set in a metric space, examples of open sets in various metric spaces. Hausdorff property. Interior of a set. Properties of open sets. Structure of an open set in R. Equivalent metrics. Distance of a point from a set, between sets, diameter of a set in a metric space and bounded sets. Closed ball in a metric space, Closed sets definition, examples. Limit point of a set, isolated point, a closed set contains all its limit points, Closure of a set and boundary of a set.

Unit II: Sequences and Complete metric spaces (15 Lectures)

Sequences in a metric space, Convergent sequence in metric space, Cauchy sequence in a metric space, subsequences, examples of convergent and Cauchy sequence in finite metric spaces, R^n with different metrics and other metric spaces.
Characterization of limit points and closure points in terms of sequences, Definition and examples of relative openness/closeness in subspaces.Dense subsets in a metric space and Separability. Definition of complete metric spaces, Examples of complete metric spaces, Completeness property in subspaces, Nested Interval theorem in R, Cantor's Intersection Theorem, Applications of Cantor's Intersection Theorem:

(i) The set of real Numbers is uncountable.
(ii) Density of rational Numbers(Between any two real numbers there exists a rational number)
(iii) Intermediate Value theorem: Let $f:[a, b] \longrightarrow R$ be continuous, and assume that f(a) and f(b) are of different signs say, $f(a) < 0$ and $f(b) > 0$. Then there exists c in (a, b) such that $f(c) = 0$.

Unit III: Compact sets (15 lectures)

Definition of compact metric space using open cover, examples of compact sets in different metric spaces R; R^2; R^n, Properties of compact sets: A compact set is closed and bounded, (Converse is not true). Every infinite bounded subset of compact metric space has a limit point. A closed subset of a compact set is compact. Union and Intersection of Compact sets.

Equivalent statements for compact sets in R:

(i) Sequentially compactness property.

(ii) Heine-Borel property: Let I be a closed and bounded interval. Let G be a family of open intervals such that I is contained in the union of members of G then I is contained in the union of a finite number of open intervals of the given family G.

(iii) Closed and boundedness property.

(iv) Bolzano-Weierstrass property: Every bounded sequence of real numbers has a convergent subsequence.

Reference books in Topology of Metric Spaces:

1. Kumaresan S.,(2005), *Topology of Metric spaces*, Narosa Publishing House, New Delhi
2. Copson E. T. ,(1996), *Metric Spaces*, Universal Book Stall, New Delhi.
3. Rudin W. , (1976) , *Principles of Mathematical Analysis*. McGraw-Hill, Auckland
4. Apostol T. , (1974) , *Mathematical Analysis*, Narosa Publishing House, New Delhi
5. Goldberg R. R., (1970), *Methods of Real Analysis,* Oxford and IBH Pub. Co., New Delhi.
6. Jain P. K. , Ahmed K. , (1996), *Metric Spaces*. Narosa Publishing House, New Delhi
7. Somasundaram D., Choudhary B.,(1996), *A First Course in Mathematical Analysis.* Narosa, New Delhi
8. Simmons G.F., (1963), *Introduction to Topology and Modern Analysis*, McGraw-Hi, New York.
9. Sutherland W.(1975), *Introduction to Metric and Topological Spaces*, Oxford University Press.

Additional References:

Expository articles of MTTS programme on MTTS website.

Course: Number Theory and its applications I   Course Code: SIUSMAT54

Unit I. Congruences and Factorization (15 Lectures)

Review of Divisibility, Primes and The fundamental theorem of Arithmetic.

Congruences : Definition and elementary properties, Complete residue system modulo m, Reduced residue system modulo m, Euler's function and its properties, Fermat's little Theorem, Euler's generalization of Fermat's little Theorem, Wilson's theorem,
Linear congruence, The Chinese remainder Theorem, Congruences of Higher degree,
The Fermat-Kraitchik Factorization Method.

Unit II. Diophantine equations and their solutions   (15 Lectures)

The linear Diophantine equation $ax + by = c$.
The equations $x^2 + y^2 = p$; where p is a prime.
The equation $x^2 + y^2 = z^2$, Pythagorean triples, primitive solutions.
The equations $x^4 + y^4 = z^2$ and $x^4 + y^4 = z^4$ have no solutions (x, y, z) with $xyz \neq 0$.
Every positive integer n can be expressed as sum of squares of four integers.
Assorted examples :section 5.4 of Number theory by Niven- Zuckerman -Montgomery.

Unit III. Primitive Roots and Cryptography   (15 Lectures)

Order of an integer and Primitive Roots.
Basic notions such as encryption (enciphering) and decryption (deciphering), Cryptosystems, symmetric key cryptography, Simple examples such as *Shift cipher*, *Affine cipher*, *Hill's cipher*, *Vigenère cipher*.
Concept of Public Key Cryptosystem; EIGamal cryptosystem, RSA Algorithm.
An application of Primitive Roots to Cryptography.

References:

1. Niven, H. Zuckerman and H. Montgomery, (1991) *An Introduction to the Theory of Numbers,* John Wiley & Sons. Inc.
2. David M. Burton,(2010) *An Introduction to the Theory of Numbers.* Tata McGraw Hill Edition.
3. G. H. Hardy and E.M. Wright. (1981)*An Introduction to the Theory of Numbers.* The English Language Book Society and Oxford University Press, 1981.
4. Neville Robins.(2005) *Beginning Number Theory*. Narosa Publications.
5. S.D. Adhikari. (1999) *An introduction to Commutative Algebra and Number Theory*. Narosa Publishing House.
6. N. Koblitz.(2012)*A course in Number theory and Cryptography*, Springer.
7. M. Artin, (2015)*Algebra.* Prentice Hall.
8. K. Ireland, M. Rosen. (1982)*A classical introduction to Modern Number Theory*. Second edition, Springer Verlag.
9. William Stalling.(2017) *Cryptology and network security*. Seventh edition, Pearson
10. Koshy, (2002) *Number theory and applications*, Academic Press Inc.

Course SIUSMATP5A: Practicals (Based on SIUSMAT51 and SIUSMAT52)

Practicals based on SIUSMAT51

1. Evaluation of double and triple integrals.
2. Change of variables in double and triple integrals and applications
3. Line integrals of scalar and vector fields
4. Green's theorem, conservative field and applications
5. Evaluation of surface integrals
6. Stokes and Gauss divergence theorem
7. Miscellaneous theory questions on units 1, 2 and 3.

Practicals based on SIUSMAT52

1. Quotient Spaces, Orthogonal Transformations.
2. Cayley Hamilton Theorem and Applications
3. Eigen Values & Eigen Vectors of a linear Transformation/ Square Matrices
4. Similar Matrices, Minimal Polynomial, Invariant Subspaces
5. Diagonalisation of a matrix
6. Orthogonal Diagonalisation and Quadratic Forms.
7. Miscellaneous Theory Questions based on units 1, 2, 3

Course SIUSMATP5B: Practicals (Based on SIUSMAT53 and SIUSMAT54)

Practicals based on SIUSMAT53

1. Examples of Metric Spaces, Normed Linear Spaces,
2. Sketching of Open Balls in R^2, Open and Closed sets, Equivalent Metrics
3. Subspaces, Interior points, Limit Points, Dense Sets and Separability, Diameter of a set, Closure.
4. Limit Points ,Sequences , Bounded , Convergent Sequences.
5. Cauchy sequences and Complete Metric Spaces.
6. Examples of Compact Sets
7. Miscellaneous Theory Questions based on units 1,2 and 3.

Practicals based on SIUSMAT54

Use of non-programmable scientific calculator is allowed in practicals of this paper.

1. Congruences.
2. Linear congruences and congruences of Higher degree.
3. Linear diophantine equation.
4. Pythagorean triples and sum of squares.
5. Cryptosystems (Private Key).
6. Cryptosystems (Public Key) and primitive roots.
7. Miscellaneous theoretical questions based on units 1, 2, and 3.

Program: T. Y. B.Sc.
Course: Mathematics

Learning Outcomes

Semester VI

SIUSMAT61 : Basic complex analysis

This course aims to introduce students to The Complex Plane C . It is useful for the students who wish to pursue higher study in mathematics to be able to understand the similarities and differences in the set of real numbers and that of complex numbers. The course should enable students to

- Understand the Complex Plane. Basic properties of complex numbers.
- Understand the concept of an Analytic function and Cauchy Riemann equations.
- Discuss convergence of Complex series and sequences.
- Be able to compute contour integrals
- Understand and apply Cauchy's Theorem and Cauchy Integral Formula.
- Construct Taylor's series of analytic functions.
- Discuss the type of singularities of a function and use Laurent's Series to classify them.

SIUSMAT62 : Algebra

This course introduces topics of Abstract algebra,It is an extension some ideas introduced in Second year. The students learn to write proofs based on abstract ideas.The course aims to enable the students to

- Understand the concept of groups further,such as subgroups,Normal subgroups.
- Learn the proofs of standard theorems such as isomorphism theorems
- Solve the theoretical problems based on these concepts.
- Learn Ring theory,integral domains,fields.
- Learn polynomial rings and concept of irreducible elements, Einstein's criterion

SIUSMAT63 : Topology of Metric Spaces and Real Analysis

This course introduces topics of Metric spaces and also revisits some concepts of Real Analysis introduced in previous semesters. The course should enable students to

- Discuss Pointwise and uniform convergence of sequence of functions and series of functions and their properties.
- Understand the concept of Continuity in Metric Spaces
- Understand the concept of Uniform Continuity, Contraction maps, Fixed point theorem
- Understand the concept of Connected Sets , Connected Metric Spaces
- Discuss Path Connectedness, Convex sets, and understand the relation between Continuity and Connectedness

SIUSMAT64 : Number Theory and Its applications II

This course introduces students to the concept of simple continued fractions(finite and infinite), the convergence properties to reals, solution of Pell's equation
The students who successfully complete this course will be able to

- discuss results on Fermat primes, Mersenne primes
- understand the concept of pseudoprimes
- solve quadratic congruences using quadratic reciprocity
- discuss various special arithmetic functions.

SEMESTER VI				
Basic complex analysis				
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT61	I	Introduction to Complex Analysis	2.5	3
	II	Cauchy Integral Formula		
	III	Complex power series, Laurent series and isolated singularities		
Algebra				
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT62	I	Group Theory	2.5	3
	II	Ring Theory		
	III	Polynomial Rings, Field theory, Homomorphism		
Topology of Metric Spaces and Real Analysis				
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT63	I	Sequences and series of functions	2.5	3
	II	Continuous functions on Metric spaces		
	III	Connected sets		
Number Theory and Its applications II				
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT64	I	Quadratic Reciprocity	2.5	3
	II	Continued Fractions		
	III	Pell's equation, Arithmetic functions & special functions		
PRACTICALS				
SIUSMATP6A	Practicals based on Courses **SIUSMAT61, SIUSMAT62**		3	6
SIUSMATP6B	Practicals based on Courses **SIUSMAT63, SIUSMAT64**		3	6

Teaching Pattern

1. Three lectures per week per course (1 lecture/period is of 48 minutes duration).
2. One practical of three periods per week per course (1 lecture/period is of 48 minutes duration).

SEMESTER VI

Course :Basic Complex Analysis Course Code: SIUSMAT61

Prerequisites: Complex plane, polar coordinates, powers and roots of complex numbers, De Moivre's formula,

Unit I: Introduction to Complex Analysis (15 Lectures)

(i) Bounded and unbounded sets, Sketching of a set in complex plane, Complex functions, Elementary functions like e^z & z^2 and their geometric properties, point at infinity- extended complex plane (Stereographic projection).

(ii) Limit of a function at a point, theorems on limits, Sequences in C, convergence of sequences and results using properties of real sequences, Continuity of functions at a point and algebra of continuous functions.

(iii) Derivative of complex functions, comparison between differentiability in real and complex sense, Cauchy-Riemann equations, sufficient conditions for differentiability, Analytic functions, algebra of analytic functions, chain rule.

(iv) Theorem: If $f' = 0$ in a domain D, then f must be constant throughout D. Harmonic functions and harmonic conjugate.

Unit II : Cauchy Integral Formula (15 Lectures)

(i) Evaluation and basic properties of contour integral including absolute inequality and M-L inequality.

(ii) Cauchy's theorem for simply and doubly connected domains, Cauchy - Goursat theorem(only statement)

(iii) Cauchy integral formula, Extension of Cauchy integral formula for derivatives, Cauchy estimates(Cauchy Inequality), Morera's Theorem

Unit III : Complex power series, Laurent series and isolated singularities. (15 Lectures)

(i) Liouville's theorem and applications including proof of Fundamental Theorem of Algebra

(ii) Linear Fractional Transformations: definition and examples.

(iii) Power series of complex numbers and related results , radius of convergence, disc of convergence, uniqueness of series representation, examples.

(iv) Taylor's theorem for analytic function, Definition of Laurent series , Definition of isolated singularity, statement (without proof) of existence of Laurent series expansion in neighbourhood of an isolated singularity, classification of isolated singularities using Laurent series expansion, examples.

Reference Books :

1. Brown J.W. and Churchill R.V., (2004), *Complex analysis and Applications*. McGraw-Hill Higher Education.

Other References:

1. Ahlfors L. ,(1990) *Complex Analysis*, Third Edition. McGraw Hill Education.
2. Greene R. E. and Krantz S. G. , (2006), *Function theory of one complex variable,* American Mathematical Society.
3. Gamelin T.W., (2000), *Complex analysis,* Springer.

Course : Algebra Course Code: SIUSMAT62

Unit I. Group Theory (15 Lectures)

Review of Groups, Subgroups, Abelian groups, Order of a group, Finite and infinite groups, Cyclic groups, The Center $Z(G)$ of a group G, Cosets, Lagrange's theorem, Group homomorphisms, isomorphisms, automorphisms, inner automorphisms (No questions to be asked)

Normal subgroups: Normal subgroups of a group, definition and examples including center of a group, Quotient group, Alternating group A_n, Cycles. Listing normal subgroups of A_4; S_3. First Isomorphism theorem (or Fundamental Theorem of homomorphisms of groups), Second Isomorphism theorem, third Isomorphism theorem, Cayley's theorem, External direct product of a group, Properties of external direct products, Order of an element in a direct product, criterion for direct product to be cyclic, Classification of groups of order 7.

Unit II. Ring Theory (15 Lectures)

Motivation: Integers & Polynomials.

Definition of a ring (The definition should include the existence of a unity element), zero divisor, unit, the multiplicative group of units of a ring. Basic Properties & examples of rings,including $Z; R; Q; C; M_n(R); Q[X]; R[X]; C[X]; Z[i]; Z[\sqrt{2}]; Z[\sqrt{-5}]; Z_n$.

Definitions of Commutative ring, integral domain (ID), Division ring, examples. Theorem such as: A commutative ring R is an integral domain if and only if for $a; b; c \in R$ with $a \neq 0$ the relation $ab = ac$ implies that $b = c$. Definitions of Subring, examples. Ring homomorphisms, Properties of ring homomorphisms, Kernel of ring homomorphism, Ideals, Operations on ideals and Quotient rings, examples. Factor theorem and First and second Isomorphism theorems for rings, Correspondence Theorem for rings: If $f : R \rightarrow R'$ is a surjective ring homomorphism, then there is a 1- 1 correspondence between the ideals of R containing the $ker f$ and the ideals of R. Definitions of characteristic of a ring, Characteristic of an ID.

Unit III. Polynomial Rings and Field theory (15 Lectures)

Principal ideal, maximal ideal, prime ideal, the characterization of the prime and maximal ideals in terms of quotient rings. Polynomial rings, R[X] when R is an integral domain/ Field. Divisibility in Integral Domain, Definitions of associates, irreducible and primes. Prime (irreducible) elements in R [X]; Q [X]; Z_p $[X]$. Eisenstein's criterion for irreducibility of a polynomial over Z. Prime and maximal ideals in polynomial rings. Definition of field, subfield and examples, characteristic of fields. Any field is an ID and a finite ID is a field. Characterization of fields in terms of maximal ideals, irreducible polynomials. Construction of quotient field of an integral domain (Emphasis on Z; Q). A field contains a subfield isomorphic to Z_p or Q.

Recommended Books

1. P. B. Bhattacharya, S. K. Jain, and S. R. Nagpaul,(1995) *Abstract Algebra,* Second edition, Foundation Books, New Delhi, 1995.

2. N. S. Gopalakrishnan,(1994) *University Algebra*, Wiley Eastern Limited.

3. N. Herstein,(1975), *Topics in Algebra*, Wiley Eastern Limited, Second edition.

4. M. Artin, (2015),*Algebra*, Prentice Hall of India, New Delhi.

5. J. B. Fraleigh, (1998), *A First course in Abstract Algebra*, sixth edition, Narosa, New Delhi.

6. J. Gallian,(2012), *Contemporary Abstract Algebra,* Narosa, New Delhi.

Additional Reference Books:

1. S. Adhikari, (1999),*An Introduction to Commutative Algebra and Number theory,* Narosa Publishing House.

2. U. M. Swamy,,A. V. S. N. Murthy,(2011),*Abstract and Modern* Algebra, Pearson.

3. Sen, Ghosh and Mukhopadhyay, (2006),*Topics in Abstract Algebra*, Universities press

Course: Topology of Metric Spaces and Real Analysis Course Code: SIUSMAT63

Unit I : Sequence and series of functions:(15 lectures)

Sequence of functions - pointwise and uniform convergence of sequences of real- valued functions, examples. Uniform convergence implies pointwise convergence, example to show converse is not true, series of functions, convergence of series of functions, Weierstrass M-test. Examples. Properties of uniform convergence: Continuity of the uniform limit of a sequence of continuous function, conditions under which integral and the derivative of sequence of functions converge to the integral and derivative of uniform limit on a closed and bounded interval. Examples. Consequences of these properties for series of functions, term by term differentiation and integration. Power series in R centered at origin and at some point in R, radius of convergence, region (interval) of convergence, uniform convergence, term by-term differentiation and integration of power series, Examples. Uniqueness of series representation, functions represented by power series, classical functions defined by power series such as exponential, cosine and sine functions, the basic properties of these functions.

Unit II : Continuous functions (15 Lectures)

Continuous functions on metric spaces Epsilon-delta definition of continuity at a point of a function from one metric space to another. Characterization of continuity at a point in terms of sequences, open sets and closed sets and examples, Algebra of continuous real valued functions on a metric space. Continuity of composite continuous function. Continuous image of compact set is compact, Uniform continuity in a metric space, definition and examples (emphasis on R). Let (X, d) and (Y, d) be metric spaces and $f : X \longrightarrow Y$ be continuous. If (X, d) is compact metric, then $f : X \longrightarrow Y$ is uniformly continuous. Contraction mapping and fixed point theorem, Applications.

Unit III: Connected sets: (15 Lectures)

Separated sets- Definition and examples, disconnected sets, disconnected and connected metric spaces, Connected subsets of a metric space, Connected subsets of R. A subset of R is connected if and only if it is an interval. A continuous image of a connected set is connected. Characterization of a connected space, viz. a metric space is connected if and only if every continuous function from X to {1, -1} is a constant function. Path connectedness in R^n, definition and examples. A path connected subset of R^n is connected, convex sets are path connected. Connected components. An example of a connected subset of R^n which is not path connected.

Reference Books:

1. Kumaresan S., (2005), *Topology of Metric spaces*, Narosa Publishing House, New Delhi
2. Copson E. T. , (1996), *Metric Spaces*, Universal Book Stall, New Delhi.
3. Rudin W. , (1976) , *Principles of Mathematical Analysis*. McGraw-Hill, Auckland
4. Apostol T. , (1974) , *Mathematical Analysis*, Narosa Publishing House, New Delhi
5. Goldberg R. R., (1970), *Methods of Real Analysis,* Oxford and IBH Pub. Co., New Delhi.
6. Jain P. K. , Ahmed K. , (1996), *Metric Spaces*. Narosa Publishing House, New Delhi
7. Somasundaram D., Choudhary B., (1996), *A First Course in Mathematical Analysis.* Narosa, New Delhi
8. Simmons G.F., (1963), *Introduction to Topology and Modern Analysis*, McGraw-Hi, New York.
9. Sutherland W. (1975), *Introduction to Metric and Topological Spaces*, Oxford University Press.

Course: Number Theory and its applications II   Course Code: SIUSMAT64

Unit I. Quadratic Reciprocity (15 Lectures)

Quadratic residues and Legendre Symbol, Gauss Lemma, Theorem on Legendre Symbol $\left(\frac{2}{p}\right)$ and $\left(\frac{3}{p}\right)$, and associated results. If p is an odd prime and a is an odd integer with (a, p) = 1 then $\left(\frac{a}{p}\right) = (-1)^t$ where $t = \sum_{k=1}^{\frac{p-1}{2}} [\frac{ka}{p}]$. Quadratic Reciprocity law.

The Jacobi Symbol and law of reciprocity for Jacobi Symbol.
Quadratic Congruences with Composite moduli.

Unit II. Continued Fractions (15 Lectures)

Finite continued fractions. Infinite continued fractions and representation of an irrational number by an infinite simple continued fraction,
Rational approximations to irrational numbers and order of convergence,
Best possible approximations.
Periodic continued fractions.

Unit III. Pell's equation, Arithmetic functions and Special numbers (15 Lectures)

Pell's equation $x^2 - dy^2 = 1$, where d is not a square of an integer.
Solutions of Pell's equation (The proofs of convergence theorems to be omitted). Arithmetic functions of number theory: $d(n)$ (or $\tau(n)$); $\sigma(n)$; $\sigma_k(n)$; $\omega(n)$ and their properties, $\mu(n)$ and the *Möbius* inversion formula.
Special numbers: Fermat numbers, Mersenne numbers, Perfect numbers, Amicable numbers, Pseudoprimes, Carmichael numbers.

Recommended Books/ References:

1. Niven, H. Zuckerman and H. Montgomery, (1991)*An Introduction to the Theory of Numbers,* John Wiley & Sons. Inc.

2. David M. Burton,(2010)*An Introduction to the Theory of Numbers.* Tata McGraw Hill Edition.

3. G. H. Hardy and E.M. Wright. (1981)*An Introduction to the Theory of Numbers.* Low priced edition. The English Language Book Society and Oxford University Press, 1981.

4. Neville Robins.(2005) *Beginning Number Theory*. Narosa Publications.

5. S.D. Adhikari. (1999) *An introduction to Commutative Algebra and Number Theory*. Narosa Publishing House.

6. N. Koblitz.(2012) *A course in Number theory and Cryptography*, Springer.

7. M. Artin, (2015) *Algebra.* Prentice Hall.

8. K. Ireland, M. Rosen. (1982) *A classical introduction to Modern Number Theory*. Second edition, Springer Verlag.

9. William Stalling.(2017) *Cryptology and network security*. Seventh edition

10. Koshy, (2002) *Number theory and applications*,

Course Code: SIUSMATP6A Practicals (Based on SIUSMAT61 and SIUSMAT62)

Practicals based on SIUSMAT61

1. Limit continuity and derivatives of functions of complex variables
2. Stereographic Projection , Analytic function, finding harmonic conjugate
3. Contour Integral, Cauchy Integral Formula , Mobius transformations
4. Taylor's Theorem , Exponential , Trigonometric, Hyperbolic functions
5. Power Series , Radius of Convergence, Laurent's Series
6. Finding isolated singularities, removable, pole and essential, Cauchy Residue theorem.
7. Miscellaneous theory questions based on Unit 1, 2 and 3.

Practicals based on SIUSMAT62

1. Normal Subgroups and quotient groups.
2. Cayley's Theorem and external direct product of groups.
3. Rings, Subrings, Ideals, Ring Homomorphism and Isomorphism
4. Prime Ideals and Maximal Ideals
5. Polynomial Rings
6. Fields.
7. Miscellaneous Theory questions based on Unit 1, 2 and 3.

Course SIUSMATP6B: Practicals (Based on SIUSMAT63 and SIUSMAT64)

Practicals based on SIUSMAT63

- Point wise and uniform convergence of sequence of functions and properties.
- Point wise and uniform convergence of series of functions and properties.
- Continuity in Metric Spaces
- Uniform Continuity, Contraction maps, Fixed point theorem
- Connected Sets , Connected Metric Spaces
- Path Connectedness, Convex sets, Continuity and Connectedness
- Miscellaneous Theoretical questions based on unit 1, 2 and 3.

Practicals based on SIUSMAT64

1. Legendre Symbol.
2. Jacobi Symbol and Quadratic congruences with composite moduli.
3. Finite continued fractions.
4. Infinite continued fractions.
5. Pell's equations and Arithmetic functions of number theory.
6. Special Numbers.
7. Miscellaneous Theoretical questions based on unit 1, 2 and 3.

Scheme of Evaluation for Semesters V & VI

The performance of the learners shall be evaluated in three ways:

(a) Continuous Internal Assessment of 40 marks in each course in each semester.

(b) Semester End Examinations of 60 marks in each course at the end of each semester.

(c) A Practical exam of 200 marks for all the four courses at the end of each semester.

(a) Internal Assessment in each Course in each semester

Sr No	Evaluation type	Marks
1	One class test	20
2	Presentation/ Project /Assignment	10
3	Viva Voce	10
Total		40

(b) Semester end examination in each course at the end of each semester (60 marks)

Duration – 2 hours .

Question Paper Pattern:- Four questions each of 15 marks.

Question Nos 1, 2 and 3 will be on unit I, II, III respectively.

Question 4 will be based on entire syllabus.

All questions shall be compulsory with not more than 50% internal choice within the questions. Question may be subdivided into sub-questions a, b, c.

(c) Practical Examination of 100 marks in each course at the end of each semester

Practical Course	Sem	Part A	Parts B	Marks out of	Duration	Journal	Viva
SIUSMATP5A	V	Questions on SIUSMAT51	Questions on SIUSMAT52	80	3 Hours	10 marks	10 marks
SIUSMATP5B	V	Questions on SIUSMAT53	Questions on SIUSMAT54	80	3 Hours	10 marks	10 marks
SIUSMATP6A	VI	Questions on SIUSMAT61	Questions on SIUSMAT62	80	3 Hours	10 marks	10 marks
SIUSMATP6B	VI	Questions on SIUSMAT63	Questions on SIUSMAT64	80	3 Hours	10 marks	10 marks

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