AC/04.08.2018/RS1

SIES College of Arts, Science and Commerce

(Autonomous)

Affiliated to Mumbai University

Syllabus under Autonomy - July 2018

Program: S.Y. B.Sc.

Course: Mathematics

Choice Based Credit System CBCS

with effect from the academic year 2018-19

Program: S.Y. B.Sc.
Course: Mathematics

Broad Objectives

To provide a degree programme in mathematics which is intellectually challenging and rigorous and whose graduates are well placed to pursue post graduate studies or to enter employment.
On successful completion of this course, all students should

1. have learnt to apply critical and analytical reasoning and to present logical and concise arguments.
2. have developed problem solving skills
3. have covered the core topics of advanced mathematics which our Department considers appropriate to their degree programme.
4. be able to comprehend high levels of abstraction in study of pure mathematics.

Learning Outcomes

SEMESTER III

SIUSMAT31 (CALCULUS OF SEVERAL VARIABLES)

In the previous semesters, students have studied the calculus of one variable functions. In This course, they learn to extend these concept to the general Euclidean space R^n . This course should enable them to

- Discuss Convergence of sequences in R^n
- Understand the concept of neighbourhoods of points in R^n and sketch the same for subsets of R^2 and R^3 Understand the concept of Open and Closed sets in R^n and be able to classify subsets of R^2 and R^3
- Understand the concept of Scalar and Vector valued functions defined on R^n , Basic concepts like limit of a Scalar and a Vector valued function at a point, continuity of a function at a point in R^n , and be able to discuss the same for functions on R^2 and R^3
- Understand the concept of Existence of partial derivatives and directional derivatives of a scalar field at a point, Gradient, The direction of Maximum and Minimum rate of change of a scalar field, and be able to solve problem on R^2 and R^3
- Understand the concept of derivative of a scalar field in terms of a linear transformation, relation between differentiability and continuity, relation between differentiability and existence and continuity of partial derivatives, be able to discuss differentiability of real valued functions of 2 and 3 variables.
- The optimization techniques, finding maxima and minima of functions of 2 variable. Constrained Maxima Minima.
- Introduction to Vector valued functions and their derivative as a linear transformation, and also as a matrix. Relation between Jacobian and Derivative. Examples based on R^2 and R^3 .

SIUSMAT32 (ALGEBRA III)

Successful completion of this course will enable students to

- understand linear isomorphisms and apply Rank- Nullity theorem
- find rank of matrix using elementary matrices and their relation with matrix units
- understand relation between rank of a matrix and solution space of non-homogeneous system
- understand relation between determinant of a matrix and permutations, also existence and uniqueness of the determinant
- develop the application of Cramer's rule and finding inverse of a matrix using adjoint of the matrix
- find orthogonal basis of an inner product space using Gram-Schmidt orthogonalization process

SIUSMAT33 (DISCRETE MATHEMATICS)

Unlike Calculus problems, combinatorial problems are typically not solvable with a core set of theorems and formulae. Most of them are solved through a careful logical analysis of possibilities and thus the main goal of this course is the development of these combinatorial reasoning skills. On successful completion of this course the student develops the experience and confidence to try multiple approaches to problem solving.

SEMESTER III				
CALCULUS OF SEVERAL VARIABLES				
Course Code	UNIT	TOPICS	Credit s	L/Week
SIUSMAT31	I	Functions of several variables	2	3
	II	Differentiation		
	III	Applications		
ALGEBRA III				
Course Code	UNIT	TOPICS	Credit s	L/Week
SIUSMAT32	I	Linear Transformations and Matrices	2	3
	II	Determinants		
	III	Inner Product Spaces		
DISCRETE MATHEMATICS				
Course Code	UNIT	TOPICS	Credit s	L/Week
SIUSMAT33	I	Permutations and Recurrence relation	2	3
	II	Preliminary Counting		
	III	Advanced Counting		
PRACTICALS				
SIUSMATP3	Practicals based on Courses **SIUSMAT31, SIUSMAT32, SIUSMAT33**		3	5

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and examples.

SIUSMAT31: CALCULUS OF SEVERAL VARIABLES

Unit I : Functions of several variables (15 Lectures)

1. The Euclidean inner product on R^n and Euclidean norm function on R^n, distance between two points, open ball in R^n, definition of an open subset of R^n, neighbourhood of a point in R^n, sequences in R^n, convergence of sequences – these concepts should be specifically discussed for $n = 2$ and $n = 3$.
2. Functions from $R^n \to R$ (scalar fields) and from $R^n \to R^m$ (vector fields), limits, continuity of functions, basic results on limits and continuity of sum, difference, scalar multiples of vector fields, continuity and components of a vector field.
3. Directional derivatives and partial derivatives of scalar fields.
4. Sketching of Quadric surfaces, level curves, level surfaces.

Unit II : Differentiation (15 Lectures)

1. Gradient of a scalar field, geometric properties of gradient, level sets and tangent planes.
2. Chain rule for scalar fields.
3. Higher order partial derivatives, mixed partial derivatives, sufficient condition for equality of mixed partial derivative.
4. Differentiability of a scalar field(in terms of linear transformation), the total derivative, uniqueness of total derivative of a differentiable function at a point, simple examples, differentiability at a point of a function f implies continuity and existence of directional derivatives of f at the point, the existence of continuous partial derivatives in a neighbourhood of a point implies differentiability at the point.
5. Mean value theorem for derivatives of scalar fields.

Unit III : Applications (15 lectures)

1. Second order Taylor's formula for scalar fields.
2. Maxima, minima and saddle points.
3. Second derivative test for extrema of functions of two variables.
4. Method of Lagrange Multipliers.
5. Differentiability of vector fields, definition of differentiability of a vector field at a point, Jacobian matrix, differentiability of a vector field at a point implies continuity. The chain rule for derivative of a vector field (statements only).

Reference Books:

1. Apostol T. , (1969) , Second Edition, *Calculus, Vol. 2,* John Wiley, New York.
2. Stewart J. ,(2009), *Calculus with early transcendental Functions*, Cengage Learning.
3. Thomas G. B. and Finney R. L.,(1998), *Calculus and Analytic Geometry*, Ninth Ed. (ISE Reprint), Addison Wesley, Reading Mass.

Additional Reference Books

1. Ghorpade S. R. and Limaye B., (2009), *A course in Multivariable Calculus and Analysis*, Springer International Edition.
2. Howard Anton, (1999), *Calculus-A* new *Horizon*, Sixth Edition, John Wiley and Sons Inc.
3. Marsden J.E. and Tromba A.J.,(1996) (Fourth Edition), *Vector Calculus*, W.H. Freeman and Co., New York.
4. Protter M.H. and Morrey C.B. Jr., (1995), *Intermediate Calculus*, Second Ed., Springer-Verlag, New York.

Online reference: http://mtts.org.in/useful-links/

SIUSMAT32 ALGEBRA III

Note: Revision of relevant concepts is necessary.

Unit 1: Linear Transformations and Matrices (15 lectures)

1. Review of linear transformations: Kernel and image of a linear transformation, Rank-Nullity theorem (with proof), Linear isomorphisms, inverse of a linear isomorphism, Any n-dimensional real vector space is isomorphic to R^n.
2. The matrix units, row operations, elementary matrices, elementary matrices are invertible and an invertible matrix is a product of elementary matrices.
3. Row space, column space of an $m\times n$ matrix, row rank and column rank of a matrix, Equivalence of the row and the column rank, Invariance of rank upon elementary row or column operations.
4. Equivalence of rank of an $m\times n$ matrix A and rank of the linear transformation $L_A : R^n \to R^m$, $L_A(X) = AX$. The dimension of solution space of the system of linear equations $AX = 0$ equals $n-\text{rank}(A)$.
5. The solutions of non-homogeneous systems of linear equations represented by $AX = B$, Existence of a solution when rank(A) = rank(A,B), The general solution of the system is the sum of a particular solution of the system and the solution of the associated homogeneous system.

Unit II : Determinants (15 Lectures)

1. Definition of determinant as an n-linear skew-symmetric function from $R^n \times R^n \times \ldots\ldots \times R^n \to R$ such that determinant of $(E_1, E_2, \ldots, E_n)$ is 1, where E_j denotes the j^{th} column of the $n \times n$ identity matrix In. Determinant of a matrix as determinant of its column vectors (or row vectors).
2. Existence and uniqueness of determinant function via permutations, Computation of determinant of 2×2, 3×3 matrices, diagonal matrices, Basic results on determinants such as $\det(A^t) = \det(A)$, $\det(AB) = \det(A)\det(B)$, Laplace expansion of a determinant, Vandermonde determinant, determinant of upper triangular and lower triangular matrices.
3. Linear dependence and independence of vectors in R^n using determinants, The existence and uniqueness of the system $AX = B$, where A is an $n\times n$ matrix with $\det(A) \neq 0$, Cofactors and minors, Adjoint of an $n\times n$ matrix A, Basic results such as $A.\text{adj}(A) = \det(A)In$.
   An $n\times n$ real matrix A is invertible if and only if $\det(A) \neq 0$, $A^{-1} = \frac{1}{\det A} adj A$ for an invertible matrix A, Cramer's rule.
4. Determinant as area and volume.

Unit III : Inner Product Spaces (15 Lectures)

1. Dot product in R^n , Definition of general inner product on a vector space over R . Examples of inner product including the inner product $<f,g> \ = \int_{-\pi}^{\pi} f(t)\,g(t)\,dt$ on C$[-\pi,\pi]$, the space of continuous real valued functions on [- π, π].

2. Norm of a vector in an inner product space. Cauchy-Schwartz inequality, Triangle in-equality, Orthogonality of vectors, Pythagoras theorem and geometric applications in R^2 , Projections on a line, The projection being the closest approximation, Orthogonal complements of a subspace, Orthogonal complements in R^2 and R^3 . Orthogonal sets and orthonormal sets in an inner product space, Orthogonal and orthonormal bases . Gram-Schmidt orthogonalization process, Simple examples in R^3 , R^4 .

Recommended Books:

1. Lang S., (1997), *Introduction to Linear Algebra*, Springer Verlag.
2. S. Kumaresan,(2001), *Linear Algebra : A geometric approach*, Prentice Hall of India Private Limited.

Additional Reference Books:

1. M. Artin,(1991), *Algebra*, Prentice Hall of India Private Limited.
2. K. Hoffman and R. Kunze , (1978), *Linear Algebra*, Tata McGraw-Hill, New Delhi.
3. T. Banchoff and J . Wermer, (1984), *Linear Algebra through Geometry,* Springer VerlagNew york.

Online reference: http://mtts.org.in/useful-links/

SIUSMAT33: DISCRETE MATHEMATICS

Unit I: Permutations and Recurrence relation (15 lectures)

1. Permutation of objects, S_n, composition of permutations and related results, even and odd permutations, rank and signature of a permutation, cardinality of Sn, An.
2. Recurrence Relations, definition of homogeneous, non-homogeneous, linear & non-linear recurrence relation, obtaining recurrence relation in counting problems, solving(homogeneous as well as non-homogeneous) recurrence relations by using iterative methods, solving a homogeneous recurrence relation of second degree using algebraic method proving the necessary result.

Unit II: Preliminary Counting (15 Lectures)

1. Finite and infinite sets, countable and uncountable sets, examples such as $N, Z, N \times N, Q, (0,1), R$
2. Addition and multiplication Principle, counting sets of pairs, two way counting.
3. Stirling numbers of second kind. Simple recursion formulae satisfied by $S(n,k)$ for $k = 1, \ldots, n$
4. Pigeonhole principle and its strong form, its applications to geometry, monotonic sequences, etc.

Unit III: Advanced Counting (15 Lectures)

1. Binomial and Multinomial Theorem, Pascal identity, examples of standard identities such as the following with emphasis on combinatorial proofs:

- $\sum_{k=0}^{r} \binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$
- $\sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}$
- $\sum_{i=0}^{k} \binom{k}{i}^2 = \binom{2k}{k}$
- $\sum_{i=0}^{n} \binom{n}{i} = 2^n$

2. Permutation and combination of sets and multisets, circular permutations, emphasis on solving problems.
3. Non-negative and positive integral solutions of equation $x_1 + x_2 + \ldots + x_n = k$
4. Principle of inclusion and exclusion, its applications, derangements, explicit formula for d_n, deriving formula for Euler's function $\phi(n)$.

Recommended Books:

1. Norman Biggs: (2002) *Discrete Mathematics*, Oxford University Press.
2. Richard Brualdi (2008) *Introductory Combinatorics*, John Wiley and sons.
3. V. Krishnamurthy(2008) *Combinatorics-Theory and Applications*, Affiliated East West Press.
4. Kenneth Rosen (2017) *Discrete Mathematics and its Applications*, Tata McGraw Hills.
5. Schaum's outline series: *Discrete mathematics*,
6. Allen Tucker (2007) *Applied Combinatorics*:, John Wiley and Sons.

SIUSMATP3: Practicals in SIUSMAT31, SIUSMAT32, SIUSMAT33

Practicals in SIUSMAT31

1. Neighbourhoods, Open and Closed sets, Sequences in R^2 and R^3.
2. Limits and continuity of scalar fields and vector fields, using definition and otherwise, iterated limits.
3. Computing directional derivatives, partial derivatives and mean value theorem of scalar fields.
4. Total derivative, gradient, level sets and tangent planes.
5. Chain rule, higher order derivatives and mixed partial derivatives of scalar fields.
6. Taylor's formula, differentiation of a vector field at a point, finding Hessian/ Jacobean matrix, Mean Value Inequality.
7. Finding maxima, minima and saddle points, second derivative test for extrema of functions of two variables and method of Lagrange multipliers.
8. Miscellaneous Theoretical Questions based on full paper.

Practicals in SIUSMAT32

1. Rank-Nullity Theorem.
2. System of linear equations.
3. Determinants, calculating determinants of 2×2 matrices, $n \times n$ diagonal, upper triangular matrices using definition and Laplace expansion.
4. Finding inverses of $n \times n$ matrices using adjoint.
5. Inner product spaces, examples. Orthogonal complements in R^2 and R^3.
6. Gram-Schmidt method.
7. Miscellaneous Theoretical Questions based on full paper

Practicals in SIUSMAT33

1. Derangement and rank,signature of permutation.
2. Recurrence relation. Formation of Recurrence relations(word problems) and solving Recurrence relations.
3. Problems based on counting principles,Two way counting.
4. Stirling numbers of second kind,Pigeonhole principle.
5. Multinomial theorem, identities, permutation and combination of multi-set.
6. Inclusion-Exclusion principle.Euler phi function.
7. Miscellaneous theory questions from all units.

Learning Outcomes
SEMESTER IV

SIUSMAT41 (INTEGRAL CALCULUS OF 1 VARIABLE)

In this course, students are introduced to the concept of Riemann Integration of a real valued function of one variable. They are aware of the technique of integration and that it is related to the process of differentiation. This course should enable them to

- Understand Integration as a process to compute area under a curve.
- Compute lower and upper sums of bounded functions on closed bounded intervals.
- Discuss integrability of a bounded function, a continuous function, and a function with finitely many discontinuities.
- Understand the relation between integrability and sum of functions, product of functions, domain additivity.
- Study Derivation of change of variables formula and integration by part.
- Understand the interdependence of integration and differentiation through Fundamental theorem of Integral Calculus.
- Study and discuss the convergence of Improper integrals, Beta and Gamma functions.
- Use integration as a tool to compute areas of bounded regions and surface area and volumes of surfaces of revolution.

SIUSMAT42 (ALGEBRA IV)

In this course students will be learning Abstract Algebra- Groups. On successful completion of this course they will

- be able to think and understand various Groups, their Subgroups, Cyclic groups.
- learn to find order of an element in the Group.
- be able to understand cosets, Lagrange's theorem, Euler's Theorem and Group Homomorphisms.

SIUSMAT43 (ORDINARY DIFFERENTIAL EQUATIONS)

Successful completion of this course will enable students to

- solve first order differential equations
- solve first and second order linear differential equations
- study and understand the solution space of differential equations
- solve application problems modelled by linear differential equations
- determine if a set of functions is linearly dependent or independent by using the Wronskian
- develop the Prey-Predator equations
- understand the use of differential equations in various disciplines of science

SEMESTER IV				
INTEGRAL CALCULUS OF 1 VARIABLE				
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT41	I	Riemann Integration	2	3
	II	Indefinite and improper integrals		
	III	Applications		
ALGEBRA IV				
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT42	I	Groups and Subgroups	2	3
	II	Cyclic Groups and Cyclic Subgroups		
	III	Lagrange's Theorem and Group homomorphism		
ORDINARY DIFFERENTIAL EQUATIONS				
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT43	I	First order First degree Differential equations	2	3
	II	Second order Linear Differential equations		
	III	Linear System of ODEs		
PRACTICALS				
SIUSMATP4	Practicals based on Courses **SIUSMAT41, SIUSMAT42, SIUSMAT43**		3	5

SIUSMAT41: INTEGRAL CALCULUS OF 1 VARIABLE

Pre-requisites: Definition of uniform continuity of real valued functions, continuity of functions on closed and bounded intervals implying uniform continuity.

Unit I: Riemann Integration (15 Lectures)

Approximation of area, Upper/Lower/General Riemann sums and properties, Upper/Lower integrals, Definition of Riemann integral on a closed and bounded interval, Criterion of Riemann integrability, If $a<c<b$ then $f \in R[a,b]$, if and only if $f \in R[a,c]$ and $f \in R[c,b]$ and $\int_a^b f = \int_a^c f + \int_c^b f$.

Properties:

$f, g \in R[a,b] \Rightarrow f + g \in R[a,b]$ and $\int_a^b f + g = \int_a^b f + \int_a^b g$

1. $f \in R[a,b]$ *and* $\lambda \in R \Rightarrow \lambda f \in R[a,b]$ and $\int_a^b \lambda f = \lambda \int_a^b f$
2. $f \in R[a,b] \Rightarrow |f| \in R[a,b]$ and $\left|\int_a^b f\right| \leq \int_a^b |f|$
3. $f \geq 0$ on $[a,b] \Rightarrow \int_a^b f \geq 0$
4. $f \in C[a,b] \Rightarrow f \in R[a,b]$
5. If f is bounded on $[a,b]$ with finite number of discontinuities then $f \in R[a,b]$
6. If f is monotone on $[a,b]$ then $f \in R[a,b]$.

Unit II: Indefinite and improper integrals (15 lectures)

1. Definition of Indefinite Riemann Integral function $F(x) = \int_a^x f(t)dt$ and its continuity.
2. 1st and 2nd Fundamental theorems of Calculus
3. Mean Value Theorem
4. Integration by parts, Change of Variable formula for integration
5. Improper integrals of types I & II, Absolute convergence, comparison tests.

Unit III: Applications (15 lectures)

1. Definition and properties of beta and gamma functions. Relationship between beta and gamma functions (without proof).
2. Applications of definite Integrals: Area between curves, finding volumes by slicing, volumes of solids of revolution-Disks and Washers, Cylindrical Shells, Lengths of plane curves, Areas of surfaces of revolution.

Reference Books:

1. Thomas G. B. and Finney R. L., (1998), *Calculus and Analytic Geometry*, Ninth Ed. (ISE Reprint), Addison Wesley, Reading Mass.
2. R. R. Goldberg, (1964), *Methods of Real Analysis*, Oxford and IBH.
3. Ajit Kumar, S. Kumaresan, (2016), *A Basic Course in Real Analysis*, CRC Press.
4. Apostol T. , (1969) , Second Edition, *Calculus, Vol. 2,* John Wiley, New York.
5. Stewart J. , (2009), *Calculus with early transcendental Functions*, Cengage, Learning.
6. Marsden J.E. and Tromba A.J., (1996) (Fourth Edition), *Vector Calculus*, W.H. Freeman and Co., New York.
7. Bartle R. G. and Sherbet D. R. , (1994), *Introduction to Real analysis*, John Wiley and sons Inc.

Online reference: http://mtts.org.in/useful-links/

SIUSMAT42: ALGEBRA IV

Unit I: Groups and Subgroups (15 Lectures)

1. Definition of a group, abelian group, order of a group, finite and infinite groups. Examples of groups including:
   a. $Z,\ Q,\ R,\ C$ under addition.
   b. $Q^*\ (= Q - \{0\})$, $R^*\ (= R - \{0\})$, $C^*\ (= C - \{0\})$, Q^+ (positive rational numbers) under multiplication.
   c. Z_n, the set of residue classes modulo *n* under addition.
   d. $U(n)$, the group of prime residue classes modulo *n* under multiplication.
   e. The symmetric group *Sn.*
   f. The group of symmetries of a plane figure. The Dihedral group D_n as the group of symmetries of a regular polygon of *n* sides (for *n*=3, 4).
   g. Klein 4-group.
   h. Matrix groups $M_{n\times n}(R)$ under addition of matrices, $GL_n(R)$, the set of invertible real matrices, under multiplication of matrices.
   i. Examples such as S^1 as subgroup of C, μ_n the subgroup of n–th roots of unity.
2. Properties such as
   1) In a group (*G*,.) the following indices rules are true for all integers *n, m.*
      i) $a^n a^m = a^{n+m}$ for all *a* in *G.*
      ii) $(a^n)^m = a^{nm}$ for all *a* in *G.*
      iii) $(ab)^n = a^n b^n$ f or all *a, b* in *G* whenever *ab=ba.*
   2) In a group $(G,.)$ the following are true:
      i) The identity element *e* of *G* is unique.
      ii) The inverse of every element in G is unique.
      iii) $(a^{-1})^{-1} = a$ for all a in G.
      iv) $(a.b)^{-1} = b^{-1}a^{-1}$ for all $a,\ b$ in G.
      v) If $a^2 = e$ for every a in *G* then $(G,.)$ is an abelian group.
      vi) $(aba^{-1})^n = ab^n a^{-1}$ for every *a, b* in *G* and for every integer *n.*
      vii) If $(a.b)^2 = a^2.b^2$ for every $a,\ b$ in *G* then $(G,.)$ is an abelian group.
      viii) $(Z_n^*,\ .)$ is a group if and only if n is a prime.
   3) Properties of order of an element such as :(*n* and *m* are integers.)
      i) If $o(a) = n$ then $a^m = e$ if and only if n/m.
      ii) If $o(a) = nm$ then $o(a^n) = m$.

iii) If $o(a) = n$ then $o(a^m) = \frac{n}{(n, m)}$ where (n, m) is the GCD of n and m.
iv) $o(aba^{-1}) = o(b)$ *and* $o(ab) = o(ba)$.
v) *If* $o(a) = m$ *and* $o(b) = m$, $ab = ba$, $(n, m) = 1$ *then* $o(ab) = nm$.

3. Subgroups
   a. Definition, necessary and sufficient condition for a non-empty set to be a Sub-group.
   b. The center $Z(G)$ of a group is a subgroup.
   c. Intersection of two (or a family of) subgroups is a subgroup.
   d. Union of two subgroups is not a subgroup in general. Union of two subgroups is a subgroup if and only if one is contained in the other.
   e. If H and K are subgroups of a group G then HK is a subgroup of G if and only if $HK = KH$.

Unit II : Cyclic groups and cyclic subgroups (15 Lectures)

1. Cyclic subgroup of a group, cyclic groups, (examples including Z, Z_n and μ_n).
2. Properties such as:
   (i) Every cyclic group is abelian.
   (ii) Finite cyclic groups, infinite cyclic groups and their generators.
   (iii) A finite cyclic group has a unique subgroup for each divisor of the order of the group.
   (iv) Subgroup of a cyclic group is cyclic.
   (v) In a finite group G, $G = < a >$ if and only if $o(G) = o(a)$.
   (vi) If $G = < a >$ and $o(a) = n$ then $G = < a^m >$ if and only if $(n, m) = 1$.
   (vii) If G is a cyclic group of order p^n and $H \subset G$, $K \subset G$ then prove that either $H \subseteq K$ or $K \subseteq H$.

Unit III : Lagrange's Theorem and Group homomorphism (15 Lectures)

1. Definition of Coset and properties such as:
   1) IF H is a subgroup of a group G and $x \in G$ then
      (i) $xH = H$ if and only if $x \in H$.
      (ii) $Hx = H$ if and only if $x \in H$.
2.
   1) If H is a subgroup of a group G and $x, y \in G$ then
      (i) $xH = yH$ if and only if $x^{-1}y \in H$.
      (ii) $Hx = Hy$ if and only if $xy^{-1} \in H$.

   2) Lagrange's theorem and consequences such as Fermat's Little theorem, Euler's theorem and if a group G has no nontrivial subgroups then order of G is a prime and G is Cyclic.

3. Group homomorphisms and isomorphisms, automorphisms

Definition. Kernel and image of a group homomorphism. Examples including inner automorphisms.

Properties such as:

(1) $f : G \rightarrow G'$ is a group homomorphism then $ker f \subset G$.

(2) $f : G \rightarrow G'$ is a group homomorphism then $ker f = \{e\}$ if and only if f is 1-1.

(3) $f : G \rightarrow G'$ is a group homomorphism then

G is abelian if and only if G' is abelian. is cyclic if and only if G' is cyclic.

Recommended Books:

1. I. N. Herstein,(1975), *Topics in Algebra*, Wiley Eastern Limited, Second edition.
2. N. S. Gopalkrishnan,(1986), *University Algebra*, Wiley Eastern Limited.
3. M. Artin, (1975), *Algebra,* Prentice Hall of India, New Delhi.
4. P.B.Bhattacharya, S.K.Jain, S.Nagpaul. (1995), *Abstract Algebra,* 2nd edi., Foundation Books, New Delhi.
5. J.B.Fraleigh, (2003), *A first course in Abstract Algebra*, 3rd edition, Narosa,New Delhi.
6. J. Gallian. (1998), *Contemporary Abstract Algebra.* Narosa, New Delhi.
7. Sharad S.Sane, (2013), *Combinatorial Techniques* Hindustan Book Agency.

Online reference: http://mtts.org.in/useful-links/

SIUSMAT43: ORDINARY DIFFERENTIAL EQUATIONS

Unit I : First order First degree Differential equations (15 Lectures)

1. Definition of a differential equation, order, degree, ordinary differential equation and partial differential equation, linear and nonlinear ODE.
2. Existence and Uniqueness Theorem for the solution of a second order initial value problem(statement only), Definition of Lipschitz function, Examples based on verifying the conditions of existence and uniqueness theorem
3. Review of Solution of homogeneous and non-homogeneous differential equations of first order and first degree. Notion of partial derivatives. Exact Equations: General solution of Exact equations of first order and first degree. Necessary and sufficient condition for $Mdx+Ndy=0$ to be exact. Non-exact equations: Rules for finding integrating factors(without proof)for non-exact equations
4. Linear and reducible linear equations of first order, finding solutions of first order differential equations, applications to orthogonal trajectories, population growth and finding the current at a given time.

Unit II : Second order Linear Differential equations(15 Lectures)

1. Homogeneous and non-homogeneous second order linear differential equations: The space of solutions of the homogeneous equation as a vector space. Wronskian and linear independence of the solutions. The general solution of homogeneous differential equations. The general solution of a non-homogeneous second order equation. Complementary functions and particular integrals.
2. The homogeneous equation with constant coefficients, auxiliary equation. The general solution corresponding to real and distinct roots, real and equal roots and complex roots of the auxiliary equation.
3. Non- homogeneous equations: The method of undetermined coefficients. The method of variation of parameters.

Unit III : Linear System of ODEs (15 Lectures)

1. Existence and uniqueness theorems.
2. Study of homogeneous and non-homogeneous linear system of ODEs in two variables.
3. The Wronskian of two solutions of a homogeneous linear system of ODEs in two variables and related results.
4. Explicit solutions of Homogeneous and non-homogeneous linear systems with constant coefficients in two variables, examples.
5. Nonlinear systems: Local existence and Uniqueness, Introduction to Volterra's prey-predator equations.

Recommended Text Books for Unit I and II:

1. G. F. Simmons, (2003) ,*Differential equations with applications and historical notes*, McGraw Hill.
2. E. A. Coddington, (1989) *An introduction to ordinary differential equations*, Dover Books.

Recommended Text Book for Unit III:

1. Hirsch, Smale & Devaney,(2003) *Differential equations, Dynamical Systems and Introduction to Chaos*, Elsevier Academic press

2. Lawrence Perko, (2012) *Differential equations and Dynamical Systems*, Springer

3. D. G. Zill, (2014) *A First Course in Differential equations and its Modelling applications*, Brooks/Cole Cengage Learning

SIUSMATP4: Practicals in SIUSMAT41, SIUSMAT42, SIUSMAT43

Practicals in SIUSMAT41

1. Calculation of upper sum,lower sum and Riemann integral.
2. Problems on properties of Riemann integral.
3. Problems on fundamental theorem of calculus, mean value theorems, integration by parts.
4. Convergence of improper integrals, applications of comparison tests.
5. Beta Gamma Functions
6. Problems on area, volume, length of arc.
7. Miscellaneous Theoretical Questions based on full paper.

Practicals in SIUSMAT42

1. Examples and properties of groups.
2. Group of symmetry of equilateral triangle, rectangle,square.
3. Subgroups.
4. Cyclic groups,cyclic subgroups,finding generators of every subgroup of a cyclic group.
5. Left and right cosets of a subgroup, Lagrange's Theorem.
6. Group homomorphisms, isomorphisms.
7. Miscellaneous Theoretical questions based on full paper.

Practicals in SIUSMAT43

1. Solving exact and non-exact equations.
2. Linear and reducible to linear equations , applications to orthogonal trajectories, population growth, and finding the current at a given time.
3. Finding general solution of homogeneous and non-homogeneous equations, use of known solutions to find the general solution of homogeneous equations.
4. Solving equations using method of undetermined coefficients and method of variation of parameters.
5. Solving second order linear ODEs
6. Solving a system of first order linear ODES.
7. Miscellaneous Theoretical questions from all units.

Teaching Pattern

1. Three lectures per week per course in each semester.
2. One practical per week per batch for each course. (5 lectures)
3. Minimum 6 practicals to be conducted in each paper in each semester.

Guidelines about conduct of Practicals

The Practicals should be conducted in batches formed as per the University circular. The Practical session should consist of discussion between the teacher and the students in which students should participate actively. The students are supposed to maintain journal for practicals.

Scheme of Evaluation for Semesters III & IV

The performance of the learners shall be evaluated in three ways:
(a) Continuous Internal Assessment of 40 marks in each course in each semester.
(b) Semester End Examinations of 60 marks in each course at the end of each semester.
(c) A combined Practical exam of 150 marks for all the three courses at the end of each semester.

(a)Internal Assessment in each Course in each semester

Sr No	Evaluation type	Marks
1	One class test	20
2	Presentation/ Project /Assignment	10
3	Viva Voce	10
Total		40

(b) Semester end examination in each course at the end of each semester (60 marks)

Duration – 2 hours .
Question Paper Pattern:- Four questions each of 15 marks.
One question on each unit (Questions 1, 2, 3).
Question 4 will be based on entire syllabus.
All questions shall be compulsory with not more than 50% internal choice within the questions. Question may be subdivided into sub-questions a, b, c.

(c)Practical Examination in each course at the end of each semester

Sr No	Evaluation type	Marks in each course			Total
1	One practical test	40	40	40	120
2	Journal for practical	10	10	10	30
Total		50	50	50	150

…………………………………………………………………………………..